WRITTEN HOMEWORK #5, DUE FEB 10, 2010

(1) (Chapter 16.9, Problem #17a) Evaluate $\iiint_E dV$, where E is the solid enclosed by the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$. Use the transformation $x = au, y = bv, z = cw$. You may want to use the fact that the determinant of a diagonal matrix is equal to the product of the elements on the diagonal.

(2) (Chapter 16.9, Problem #22) Evaluate $\iint_R \sin(9x^2 + 4y^2)\, dA$, where R is the region in the first quadrant bounded by the ellipse $9x^2 + 4y^2 = 1$. (Hint: the coordinate change you want is a slight variation on polar coordinates.)

(3) (Chapter 17.2, Problem #34) A thin wire has the shape of the part of the circle $x^2 + y^2 = a^2$ in the first quadrant. If the density function is $f(x, y) = kxy$ (k some positive constant), find the mass and center of mass of the wire.

(4) (Chapter 17.2, Problem #46) The base of a circular fence with radius $10m$ is given by $x = 10 \cos t, y = 10 \sin t$. The height of the fence at position (x, y) is given by the function $h(x, y) = 4 + 0.01(x^2 - y^2$, so the height varies from $3m$ to $5m$. What is the surface area of one side of the fence? (You can ignore the part of the question in the text about paint.)

(5) (Chapter 17.1, Problem #6) Sketch the vector field

$$\mathbf{F}(x, y) = \frac{y\mathbf{i} - x\mathbf{j}}{\sqrt{x^2 + y^2}}.$$

(6) (Chapter 17.1, Problem #26) Let $f(x, y) = \sqrt{x^2 + y^2}$. Find the gradient vector field ∇f of f and sketch it.